Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

MECHANICS.

348. Proposed by ALTON L. MILLER, Ann Arbor, Michigan.

If equilateral triangles be constructed on the sides of any triangle, their centers are the vertices of a new equilateral triangle. Show that the center of gravity of this new equilateral triangle coincides with the center of gravity of the original triangle.

349. Proposed by S. A. COREY, Albia, Iowa.

A 9 pound weight is attached to a string which passes over a smooth fixed pulley. The other end of the string is fastened to and supports a smooth pulley P_1 of weight 1 pound, over which passes a second string to one end of which is attached a 3 pound weight, and the other end of which is attached to and supports another smooth pulley P_2 of weight 1 pound. Over the pulley P_2 passes a third string supporting weights, 2 pounds and $3\frac{1}{3}$ pounds.

If the system is acted on by gravity alone show that the accelerations of the 9 pound weight, $3\frac{1}{3}$ pound weight, and pulley P_2 are 0, $\frac{1}{2}g$, and $\frac{1}{3}g$, respectively.

Determine the motion of the weights when pulleys are not smooth, that is, when friction is present.

NUMBER THEORY.

266. Proposed by J. L. RILEY, Tahlequah, Oklahoma.

In how many ways can a given number be polygonal?

267. Proposed by C. C. YEN, Tangshan, North China.

A number theory function $\phi(n)$ is defined for every positive integer n, and for every such number n, it satisfies the relation $\phi(d_1) + \phi(d_2) + \cdots + \phi(d_\tau) = n$. From this property alone show that

$$\phi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\cdots\left(1 - \frac{1}{p_k}\right),$$

where $p_1, p_2, p_3, \cdots, p_k$ are the different prime factors of n.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

471. Proposed by E. T. BELL, University of Washington.

If there is an infinite number of positive integers r for which the equation $\sum_{i=1}^{n} a_i^r = \sum_{i=1}^{m} b_i^r$ holds, where the a_i and b_i are given positive integers, prove that $m = n$, and that in some order the a_i are identical with the b_i.

472. Proposed by E. T. BELL, University of Washington.

If a_i and b_i $(i = 1, \cdots, n;\ j = 1, \cdots, m)$ denote positive integers, and if $\sum_{i=1}^{n} a_i^r = \sum_{j=1}^{m} b_j^r$ for all odd positive integral values of r, prove that $m = n$, and that in some order the a_i are identical with the b_j.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

As the proof which I shall give covers both these problems, I shall not treat them separately.

Let a_1 be that a which is the largest of the a's and let there be k a's equal in value to a_1. Similarly b_1 is the largest b, and there are k' b's of the same value. (These exist since there is only a finite number of the a's and of the b's.) Now it is a well-known theorem that

$$\lim_{r \doteq \infty} \Sigma a_i^r / k a_1^r = 1.$$

Since there is an infinite number of positive integers r for which the given equations hold, we can find an r as large as we please for which they hold. For such an r, $\Sigma a_i^r = k a_1^r + \delta k a_1^r$ and

$\Sigma b_i{}^r = k'b_1{}^r + \delta'k'b_1{}^r$, where δ and δ' can be made as small in absolute value as we please by taking r sufficiently large. Since the left-hand sides of these equations are equal, the right-hand sides must be equal also. Equating them we obtain

$$\frac{k}{k'} = \frac{b_1{}^r}{a_1{}^r}\frac{(1+\delta')}{(1+\delta)} = \frac{b_1{}^r}{a_1{}^r}(1+n),$$

where n is an infinitesimal. If, now, a_1 and b_1 are not equal we can make the right-hand side as small (or as large) as we please by taking r sufficiently large. This is impossible for the ratio $k \div k'$ is a definite number. Consequently $a_1 = b_1$, and $k = k'$. In each equation we may now subtract from each side the a's and b's proven equal and proceed as before to prove the largest remaining a's and b's to be equal in value and number. We may continue in this way until there are no a's (or b's) left, when the b's (a's) must also be exhausted.

471 was also solved by C. F. GUMMER and FRANK IRWIN.

473. Proposed by J. I. GINSBURG, Student, Cooper Union, New York.

Factor the expression $x^{30} + x^{25} + x^{20} + x^{15} + x^{10} + x^5 + 1$.

SOLUTION BY CLARIBEL KENDALL, University of Colorado.

Multiplying the given expression by $x^5 - 1$, we obtain $x^{35} - 1$. This may be factored in the following way:

First, consider

$$(1)\qquad \begin{aligned} x^{35} - 1 &= (x^5)^7 - 1 = (x^5 - 1)(x^{30} + x^{25} + x^{20} + x^{15} + x^{10} + x^5 + 1) \\ &= (x - 1)(x^4 + x^3 + x^2 + x + 1)(x^{30} + x^{25} + x^{20} + x^{15} + x^{10} + x^5 + 1). \end{aligned}$$

Next consider

$$(2)\qquad \begin{aligned} x^{35} - 1 &= (x^7)^5 - 1 = (x^7 - 1)(x^{28} + x^{21} + x^{14} + x^7 + 1) \\ &= (x - 1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)(x^{28} + x^{21} + x^{14} + x^7 + 1). \end{aligned}$$

The factors of $x^{35} - 1$ must be the same in both (1) and (2). $x - 1$ occurs in both. The prime factor $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ in (2) must also be found in (1). It cannot be contained in the prime factor $x^4 + x^3 + x^2 + x + 1$ and hence must be found in the third factor. By division the factors of

$$x^{30} + x^{25} + x^{20} + x^{15} + x^{10} + x^5 + 1$$

are found to be

$$x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$$

and

$$x^{24} - x^{23} + x^{19} - x^{18} + x^{17} - x^{16} + x^{14} - x^{13} + x^{12} - x^{11} + x^{10} - x^8 + x^7 - x^6 + x^5 - x + 1.$$

Also variously solved by H. C. FEEMSTER, HORACE OLSON, O. S. ADAMS, W. F. SHELTON, J. W. BALDWIN, A. W. SMITH, E. B. ESCOTT, L. C. MATHEWSON, PAUL CAPRON, E. J. OGLESBY, and the PROPOSER.

GEOMETRY.

493. Proposed by FLORENCE P. LEWIS, Goucher College, Baltimore, Md.

Construct three circles each of which shall be tangent to the other two and to two sides of a given triangle.

NOTE BY J. L. COOLIDGE, Harvard University.

It is always a pleasure to see this old friend. Few problems in elementary geometry have a longer or more distinguished pedigree. At least forty articles dealing with it were published in the nineteenth century and the twentieth is doing its share.[1] The history is briefly this. The

---

[1] SIMON, *Die Entwickelung der Elementargeometrie im XIX Jahrhundert*, Leipzig, 1906, pp. 147ff.